Journal of Applied Mechanics and Technical Physics, No. 4, 1994

ACOUSTIC RESONANCE IN TURBOMACHINERY WITH AERODYNAMIC INTERACTION OF THE CASCADES IN SUBSONIC GAS FLOW

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UDC 621.515:534

The urgency of the need for abatement of the vibroacoustic activity of turbomachinery (pumps, turbines, compressors, etc.) is well known [1-7]. One way to solve this problem is to lower the intensity of an ever-present time-periodic source of disturbance in turbomachinery, i.e., aerodynamic interaction between the impeller and the guide-vane (nozzle) assembly. It has been shown [4-7] that vibroacoustic activity is strongly influenced by the ratio of the numbers of rotating and stationary blades. In particular, it has been established [4-6] that the unsteady forces and torques generated by the indicated interaction, which induce pressure pulsations, and also the longitudinal, torsional, and transverse vibrations of the casing and impeller of turbomachinery can be substantially reduced by finding auspicious combinations of the blade counts of the impeller and guide-vane or nozzle assembly. However, it is important to note that the relations used to select the blading combinations in [4-7] have been derived without regard for wave processes in the casings or ducting of the turbomachinery. On the other hand, several authors [2, 4, 8-10] have indicated the possibility of a considerable increase in the vibroactivity associated with aerodynamic interaction of the blade cascades of turbomachinery in the event of acoustic resonance in the gas flow in the interblade and discharge channels.

Here, using elementary mathematics, we propose to compare the conditions for the onset of acoustic resonance in a gas flow in the interblade channels of the stationary cascade as they affect the choice of combination of numbers of moving and stationary blades with the analogous conditions for the onset of hydrodynamic disequilibrium, interpreted as the presence of transient periodic forces and torques acting on the stationary vanes of a turbomachine at the impeller blade frequencies with aerodynamic interaction of the cascades. We derive a relation for estimating the maximum ratio of the number of moving to stationary blades and the condition whereby acoustic pressure oscillations in harmonics of the impeller blade frequency are unamplified simultaneously in the gas flow in the interblade channels of the stationary cascade and in the discharge outlet of an axial-flow turbomachine. We demonstrate theoretically and confirm experimentally the possibility of the generation of pressure pulsations and, as a consequence, casing vibrations of turbomachinery in the presence of interaction of rotational pressure modes with the vanes of the stationary cascade in rotational harmonics of these modes that are multiples of the number of stationary vanes.

1. An experimental investigation of acoustic resonance in the interaction of blade cascades and a theoretical determination of the conditions for its onset are reported in a study [8] of two annular cascades having a common symmetry axis, where one cascade rotates about this axis with angular velocity Ω. To determine the velocity potential of acoustic disturbances having the spectrum of the impeller blade frequencies ($\lambda_{Bn} = nB\Omega$, where $n = \pm 1, \pm 2, \ldots$, and B is the number of blades on the impeller), Izmailov et al. [8] use the solution of the inhomogeneous wave equation with variable coefficients subject to homogeneous Neumann conditions on the blade surfaces and to the radiation condition at infinity [11]. The expression derived in [8] for the amplitude function of the velocity potential of acoustic disturbances with the frequency λ_{Bn} has the form

$$\varphi_n = \sum_{p=1}^{\infty}\sum_{m=1}^{V-1}\frac{c_{pm}^{(n)}\psi_{pm}}{\lambda_{Bn}^2 - (k_{pm}\,a/b)^2}, \qquad (1.1)$$

Sergiev Posad, Moscow Oblast. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, No. 4, pp. 78-85, July-August, 1994. Original article submitted September 22, 1993.

where V is the number of stationary vanes, ψ_{pm} and k_{pm} are the eigenfunctions and eigenvalues of the problem, respectively, a is the freestream sound velocity, b is a characteristic length of the blades, and $c_{pm}^{(n)}$ denotes the coefficients of the series expansion in eigenfunctions of the function describing the physical conditions of impenetrability at the stationary vanes in the presence of aerodynamic interaction of the cascades [10, 12].

It has been shown [8] that the only nonvanishing terms of the series in (1.1) are those which satisfy the condition

$$0 < m = nB + jV < V, n, j = 0, \pm 1, \ldots \tag{1.2}$$

It follows from Eqs. (1.1) and (1.2) that acoustic resonance of the gas flow through the stationary cascade as a result of cascade interaction can occur only when

$$\lambda_{Bn} = nB\Omega = \omega^*_{pm}.$$

Here $\omega^*_{pm} = \mathrm{Re}(k^*_{pm}\, a/b)$ is the natural frequency of the gas oscillations for the corresponding eigenfunction satisfying condition (1.2), and $\mathrm{Re}(k^*_{pm}\, a/b)$ is the real part of the bracketed expression.

2. It is instructive to compare relation (1.2) with the previously derived [4] conditions for the excitation of transient forces and torques acting on the vanes of a guide-vane assembly in harmonics of the impeller blade frequency.

The following condition for the excitation of a transient torque has been obtained in [4]:

$$\frac{nB}{V} = s_1 \tag{2.1}$$

(s_1 is a positive definite integer).

From relation (2.1) we deduce

$$nB = s_1 V. \tag{2.2}$$

Substituting Eq. (2.2) into the left-hand side of inequality (1.2), we have

$$0 < m = nB + jV = s_1 V + jV = V(s_1 + j)\ (j = 0, \pm 1, \pm 2, \ldots) \tag{2.3}$$

It is evident from (2.3) that $V(s_1 + j) > V$.

Consequently, despite the presence of a transient driving torque, acoustic resonance of the gas flow in the interblade channels of the stationary cascade does not occur.

It is also noted in [4] that, when the numbers of moving and stationary vanes are chosen to satisfy the inequality

$$\frac{nB}{V} \neq s_1, \tag{2.4}$$

the net transient torque acting on the casing of the turbomachine is equal to zero.

Inequality (2.4) can be written in the form

$$\frac{nB}{V} = s_2 + \frac{m_1}{V}, \tag{2.5}$$

where $0 < m_1 < V$, and s_2 and m_1 are positive definite integers.

Equation (2.5) can be used to transform the left-hand side of inequality (1.2):

$$nB + jV = s_2 V + m_1 + jV = (s_2 + j)V + m_1. \tag{2.6}$$

It follows from (2.6) that, if we choose $j = -s_2$, then

$$nB + jV = m_1 < V. \tag{2.7}$$

Thus, when the combination of numbers of moving and stationary vanes is chosen in compliance with [4] for elimination of the driving torque, acoustic resonance of the gas flow in the interblade channels of the stationary cascade is possible according to (2.7).

The condition for the generation of the total transient driving force acting on the casing of a turbomachine has been obtained in [4]:

$$\frac{nB \pm 1}{V} = s_1 \ (\text{integer } s_1 > 0). \tag{2.8}$$

Relation (2.8) permits the left-hand side of inequality (1.2) to be written as follows:

$$nB + jV = s_1V + jV \mp 1 = (s_1 + j)V \mp 1. \tag{2.9}$$

It is evident from (2.9) that, for example, when the right-hand side of this inequality is equal to $(s_1 + j)V - 1$, it is sufficient to choose $j = -s_1 + 1$ in order for its left-hand side to be equal to $nB + jV = V - 1 < V$. When the right-hand side is equal to $(s_1 + j)V + 1$, we set $j = -s_1$ and obtain $nB + jV = 1 < V$.

Consequently, acoustic resonance is possible in the presence of a nonzero net driving force acting on the casing of the turbomachine.

We now investigate the condition obtained in [4] for the nongeneration of a finite net transient aerodynamic force acting on the turbomachine casing:

$$\frac{nB \pm 1}{V} \neq s_1,$$

which can be written in the form of an equation

$$\frac{nB \pm 1}{V} = s_2 + \frac{m_1}{V}, \tag{2.10}$$

where $0 < m_1 < V$, and s_2 and m_1 are integers.

We transform the left-hand side of inequality (1.2) in accordance with (2.10):

$$nB + jV = Vs_2 + m_1 + 1 \mp jV = (s_2 + j)V + m_1 \mp 1. \tag{2.11}$$

Equation (2.11) clearly shows that it is sufficient to choose $j = -s_2$ in order for the left-hand side of (1.2) to be equal to $nB + jV = m_1 \mp V$.

Inasmuch as $m_1 < V$, we therefore have $m_1 - 1 < V$.

Consequently, as in the choice of blade combination to ensure the absence of a net driving torque according to [4], acoustic resonance is possible when the combination of numbers of moving and stationary blades is chosen in accordance with [4] for the absence of a net driving force acting on the turbomachine casing.

3. A model of the aerodynamic interaction of blades of the moving and stationary cascades of turbomachinery is also discussed in an investigation [9] of the propagation of pressure waves generated by the interaction of blades in a cylindrical discharge channel of an axial-flow turbomachine. In [9] an expression is derived for the pressure pulsations in the interaction of an impeller having B blades with a single stator vane of a turbomachine. We note that the model can be used in this case to describe the propagation of pressure waves in the discharge channel of a centrifugal turbomachine, for example, when such waves are generated by the interaction of the impeller with the discharge tongue in a centrifugal pump with a bladeless diffuser. The indicated expression has the form

$$p(\theta, t) = \sum_{n=1}^{\infty} \sum_{s=-\infty}^{\infty} a_{sn} \cos(s\theta - nB\Omega t + \Phi_{sn}) \tag{3.1}$$

(Φ_{sn} is the initial phase of the oscillations).

The following relation for the pressure pulsations at a point situated on the axis of an axial-flow turbomachine with V vanes in the stationary cascade ($V > 1$) and B blades in the moving cascade has been obtained [9] by summation, at the given point, of the pressure pulsations propagating from all V stationary vanes interacting with the impeller blades:

$$p_{sm} = \sum_{q=0}^{V-1} a_{sn}\cos\,[s(\theta - q\Delta\theta) - nB\Omega(t - q\Delta t) + \Phi_{sn}]. \tag{3.2}$$

Here $\Delta\theta = 2\pi/V$ is the pitch of the stationary cascade, and $\Delta t = \Delta\theta/\Omega = 2\pi/V\Omega$. Carrying out the summation in (3.2), we have

$$\begin{aligned} p_{sn} = \Bigg\{ & \left\{ \frac{1}{2} + \frac{\sin\left[\left(V - \frac{1}{2}\right)\left(\frac{2\pi nB}{V} - \frac{2\pi s}{V}\right)\right]}{2\sin\left[\frac{1}{2}\left(\frac{2\pi nB}{V} - \frac{2\pi s}{V}\right)\right]} \right\} \cos(s\theta - nB\Omega t + \Phi_{sn}) \\ & - \left\{ \frac{\cos\left[\frac{1}{2}\left(\frac{2\pi nB}{V} - \frac{2\pi s}{V}\right)\right] - \cos\left[\left(V - \frac{1}{2}\right)\left(\frac{2\pi nB}{V} - \frac{2\pi s}{V}\right)\right]}{2\sin\left[\frac{1}{2}\left(\frac{2\pi nB}{V} - \frac{2\pi s}{V}\right)\right]} \right\} \sin(s\theta - nB\Omega t + \Phi_{sn}) \Bigg\} a_{sn}. \end{aligned} \tag{3.3}$$

It is evident from relation (3.3) that $p_{sn} = 0$ for $2\pi(nB - s)/V \neq 2\pi k$ ($k = 0, \pm 1, \pm 2, \ldots$).

For $2\pi(nB - s)/V = 2\pi k$, passing to the limit in (3.3), we obtain

$$p_{sn} = Va_{sn}\cos(s\theta - nB\Omega t + \Phi_{sn}).$$

Consequently, the resultant pressure pulsation on the axis of an axial-flow turbomachine has a nonzero value under the condition

$$s = nB + kV. \tag{3.4}$$

In estimating the resultant pressure pulsation in the cylindrical discharge channel of an axial-flow turbomachine at an off-axis point, we must take into account the phase shift of the pressure oscillations in acoustic waves arriving at the indicated point from the stationary vanes of the guide-vane assembly that interact with the impeller blades.

Let the point M be situated in the discharge channel 1 (see Fig. 1) at a distance ρ from the center O of the guide-vane assembly 2; the radius vector OM drawn from the center of the guide-vane assembly forms an angle ψ with the axis of the impeller 3 of the turbomachine. Considering the right triangle MNA_q, in which $\angle N$ is a right angle, we find that the path traversed by the acoustic wave from the q-th guide vane to the point M is given by the expression

$$MA_q = \sqrt{\rho^2 + R^2 - 2R\rho\sin\psi\cos\theta},$$

where R is the outside radius of the guide-vane assembly.

Consequently, in determining the resultant pressure pulsation in the n-th harmonic of the impeller blade frequency, we must take into account the phase angle

$$\alpha_q = MA_q(nB\Omega)/a = \sqrt{\rho^2 + R^2 - 2R\rho\sin\psi\cos\theta}\,\frac{nB\Omega}{a}. \tag{3.5}$$

Substituting Eq. (3.5) as additional terms in the arguments of the trigonometric functions in Eq. (3.2), we can determine the resultant pressure pulsation at any — not necessarily axial — point M of a specific turbomachine by carrying out the numerical summation in Eq. (3.2).

We now consider the important practical case where it is required to determine the pressure pulsations in the peripheral region of the plane directly adjacent to the outlet of the guide-vane assembly of a turbomachine. This case corresponds to an angle $\psi \approx \pi/2$ and $\rho = R$.

Equation (3.5) now has the form

$$\alpha_q = \sqrt{2}\,R\sqrt{1 - \cos\theta}\,\frac{nB\Omega}{a} = \sqrt{2}\,R\sqrt{2}\sin\left(\frac{\theta}{2}\right)\frac{nB\Omega}{a} = 2R\frac{nB\Omega}{a}\sin\frac{\theta}{2}.$$

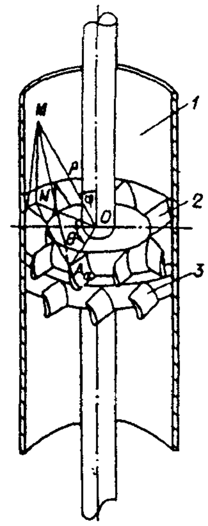

Fig. 1

To eliminate resonance amplification of the pressure oscillations at the point M as a result of interaction between acoustic waves arriving from two contiguous guide vanes q and q + 1, the difference in the phase angles $\Delta\alpha_q = \alpha_{q+1} - \alpha_q$ must be smaller than 2π. This condition is written

$$\begin{aligned}\Delta\alpha_q &= 2R\frac{nB\Omega}{a}\left[\sin\frac{\theta + \Delta\theta}{2} - \sin\frac{\theta}{2}\right] \\ &= 4R\frac{nB\Omega}{a}\sin\frac{\Delta\theta}{4}\cos\frac{2\theta + \Delta\theta}{4} < 2\pi.\end{aligned} \tag{3.6}$$

The argument of the cosine in Eq. (3.6) satisfies the inequality

$$0 < \frac{2\theta + \Delta\theta}{4} < \pi.$$

Thus, at $\theta = 0$

$$\frac{2\theta + \Delta\theta}{4} = \frac{2\pi}{4V} = \frac{\pi}{2V} < \pi.$$

and at $\theta = (V - 1)\Delta\theta$

$$\frac{2\theta + \Delta\theta}{4} = \frac{(2V - 1)\Delta\theta}{4} = \frac{(2V - 1)2\pi}{4V} = \pi\left(1 - \frac{1}{2V}\right) < \pi.$$

Hence, $\Delta\alpha_q$ is a monotonically decreasing function of its argument θ.

This property enables us to transform inequality (3.6) as follows:

$$4R\frac{nB\Omega}{a}\sin\frac{\Delta\theta}{4}\cos\frac{(2V - 1)\Delta\theta}{4} < 4R\frac{nB\Omega}{a}\sin\frac{\Delta\theta}{4} < 4R\frac{nB\Omega}{a}\frac{\Delta\theta}{4} = \frac{RnB\Omega}{a}\frac{2\pi}{V}.$$

Thus, Eq. (3.6) is clearly satisfied if the following inequality holds:

$$\frac{2\pi RnB\Omega}{aV} < 2\pi,$$

from which we deduce

$$\frac{Rn\Omega}{a}\frac{B}{V} < 1.$$

TABLE 1

Ω_e, rad/s	Ω_{Vs}, rad/s	n	k	Ω_e, rad/s	Ω_{Vs}, rad/s	n	k
2821	2840	1	1	3569	3519	2	−3
2482	2463	1	3	3688	3695	2	−4
2287	2309	1	4	3801	3889	2	−5
2187	2174	1	5	3996	4103	2	−6
2086	2053	1	6	4398	4348	2	−7
1841	1847	1	8	4744	4618	2	−8
1483	1478	1	13	4882	4926	2	−9
3311	3358	2	−2				

Consequently, to eliminate resonance amplification of the pressure oscillations in harmonics of the impeller blade frequency, the maximum ratio of the number of moving and stationary blades must satisfy the condition

$$\frac{nB}{V} < \frac{a}{R\Omega}. \tag{3.7}$$

Relation (3.7) admits the following interpretation. Let us rewrite it in the form

$$\frac{nB\Omega}{a}\frac{R}{V} = \frac{2\pi R}{V}\frac{nB\Omega}{2\pi a} = \frac{\tau_V}{L} < 1, \tag{3.8}$$

where τ_V is the pitch of the stationary cascade, and L is the wavelength of the pressure oscillations in the n-th harmonic of the impeller blade frequency.

It follows from (3.8) that the wavelength of the pressure oscillations must be greater than the blading pitch of the guide-vane assembly in order to eliminate resonance amplification of the pressure oscillations in the discharge section at the outlet of the guide-vane assembly in a region far from the axis of the turbomachine.

We have established the fact that, when the condition $nB/V = s_1$ is satisfied (s_1 is a positive integer, $s_1 \geq 1$), acoustic resonance of the gas flow in the interblade channels of the stationary cascade of a turbomachine does not occur, even though (according to [4]) vibrations and pressure pulsations in harmonics of the impeller blade frequency are generated by the net transient torque acting on the stationary vanes. The combined analysis of this condition and inequality (3.7) shows that the following relation must be satisfied if the resonance amplification of pressure oscillations is to be eliminated simultaneously in the discharge channel of an axial-flow turbomachine and in the interblade channels of the stationary cascade:

$$s_1 = \frac{nB}{V} < \frac{a}{R\Omega}. \tag{3.9}$$

It follows from inequality (3.9) that

$$M_u < 1/s_1 < 1, \tag{3.10}$$

where $M_u = R\Omega/a$ is the angular (circumferential) Mach number.

Inequality (3.10) coincides with the previously obtained [9] condition for the nontransmission of pressure waves in harmonics of the impeller blade frequency in the discharge channel of an axial-flow turbomachine.

Consequently, to eliminate the simultaneous amplification of pressure oscillations in the interblade channels of the stationary cascade and in the discharge channel of an axial-flow turbomachine, it is necessary that acoustic waves not propagate in the discharge channel.

4. It is important to note that [9] acoustic waves generated by interaction of the moving and stationary cascades can be interpreted in accordance with Eq. (3.1) as the superposition of tangential pressure modes rotating with an angular velocity $\Omega_s = nB\Omega/s$, where s is the number of pressure maxima on a circle of diameter equal to the diameter of the outlet from the guide-vane assembly. The interaction of these rotating pressure modes with the stationary vanes will obviously induce acoustic waves with a frequency $nB\Omega = s\Omega_s = s(nB\Omega)/s$. On the other hand, the stationary vanes apply a periodic perturbation to the rotating modes with a frequency $\Omega_{Vs} = lV(nB\Omega)/s$ (l is a positive definite integer), as becomes clear when we transform to a coordinate system rotating with frequency Ω_s.

The above-mentioned perturbation leads to the generation of pressure oscillations at frequencies Ω_{Vs}, which will necessarily be reflected in the spectrum of the pressure pulsations and, as a result, in the spectrum of the turbomachine structural vibrations induced by these pulsations. This fact has been confirmed in experimental studies of a centrifugal pump with a bladeless diffuser and a 12-blade impeller. During the operation of the pump the impeller blades interact with the discharge tongue, which functions as a solitary stationary vane in the given situation.

The foregoing is evidenced by the presence of discrete components at the frequencies Ω_{Vs} in a vibration spectrogram obtained in tests of the given pump.

Table 1 shows the frequencies Ω_e corresponding to the experimentally obtained discrete components, along with the frequencies Ω_{Vs} calculated from the equation

$$\Omega_{V_s} = lV(nB\Omega)/s$$

(for $\Omega = 2\pi \cdot 490$ rad/s, V = 1, B = 12, and l = 1) and the integer values of the parameters n and k in the expression (3.4) used to determine s.

It is evident from the table that the experimental and calculated frequencies of the discrete components of the vibration spectrum exhibit satisfactory agreement. We note that the frequency range of the discrete components spans frequencies both above and below the impeller frequency (3078 rad/s). The experimental vibration amplitudes at the discrete frequencies indicated in the table range from 0.81 to 1.13 times the vibration amplitude at the impeller frequency, suggesting that the level of these vibrations is quite high.

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